FAST FULL-WAVE METHODS FOR APERIODIC ANTENNA ARRAYS FOR SPACE APPLICATIONS

PhD Defense, Magnus Brandt-Møller, DTU Space

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PROJECT OVERVIEW

Fast Full-wave Methods for Aperiodic Antenna Arrays for Space Applications

Study period	2020-08-15 to 2023-12-06	**Committee**	Prof. Mats Gustafsson
Supervisors (Uni.)	Michael Mattes & Olav Breinbjerg		Prof. Matthys Botha
Supervisors (Com.)	Min Zhou & Erik Jørgensen	**Chairperson**	Prof. Jørgen Dall

Part I — Introduction

Part II — Array Decomposition Method with Higher-Order Basis Functions

Part III — Connected & Aperiodic Array Antennas

Part IV — Non-Identical & Non-All-Metal Arrays

DISPOSITION

Part I — **Introduction**

- Background & Motivation
- Antenna Arrays for Space Applications
- Research Objectives
- Towards new State-of-the-Art

Part II — **Array Decomposition Method with Higher-Order Basis Functions**

Part III — **Connected & Aperiodic Array Antennas**

Part IV — **Non-Identical & Non-All-Metal Arrays**

DISPOSITION

BACKGROUND & MOTIVATION

Established
Trend

Global TV
Remote Sensing
Deep-Space exploration
Navigation Systems

GEO

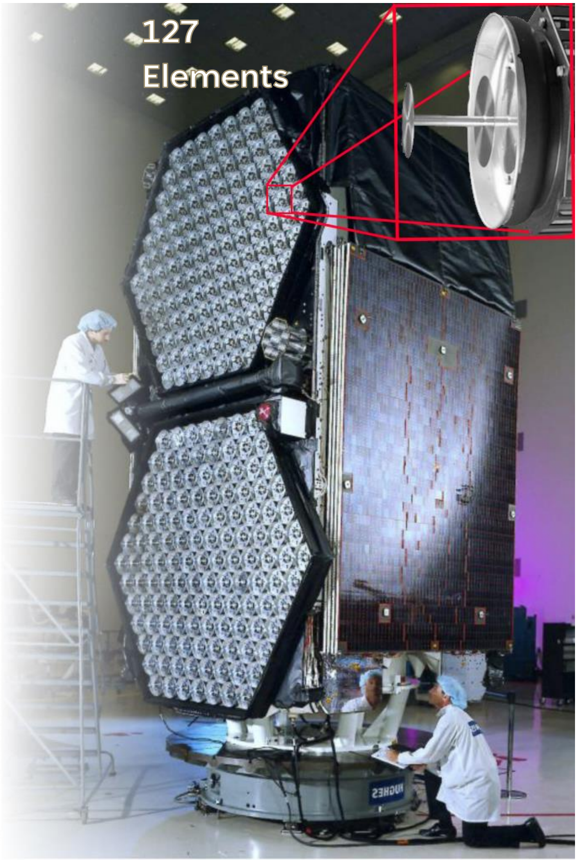
LEO

Emerging
Trend

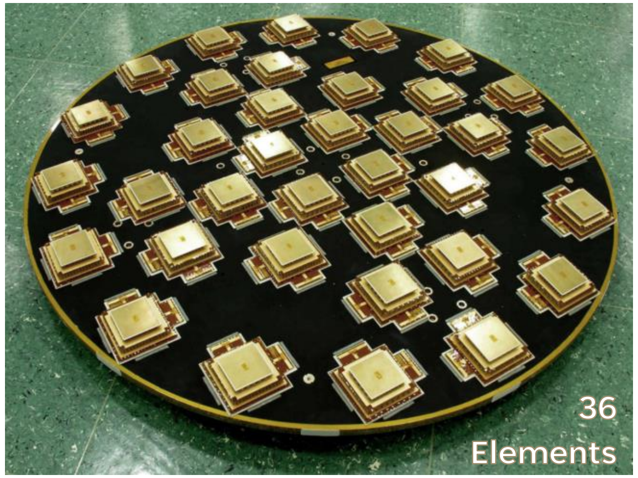
ANTENNA ARRAYS FOR SPACE APPLICATIONS (1/2)

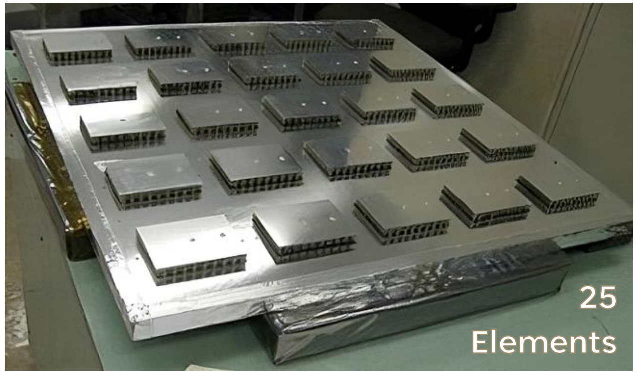
The Beginning

ESA GIOVE-A (2005)

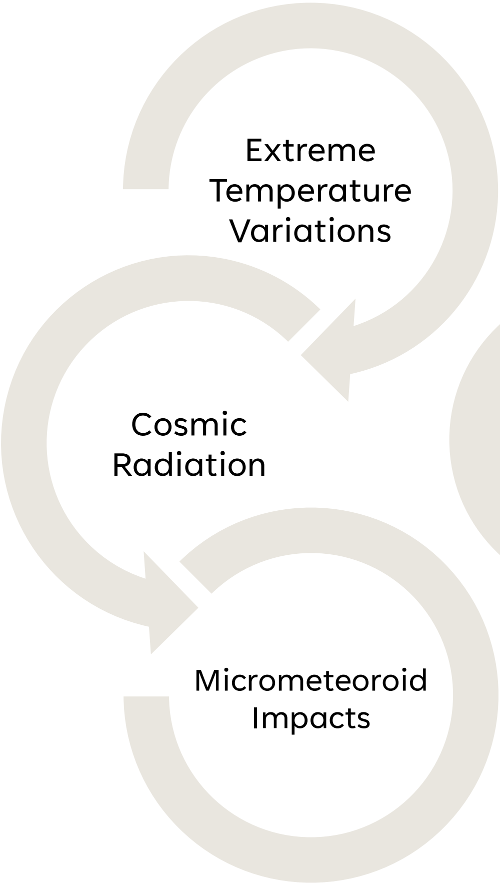


HUGHES ICO F-1 (2000)





NASA JUNO (2011)



THE EMPTY YET EXTREME
REALM OF SPACE

All Metal

High Radiation Efficiency

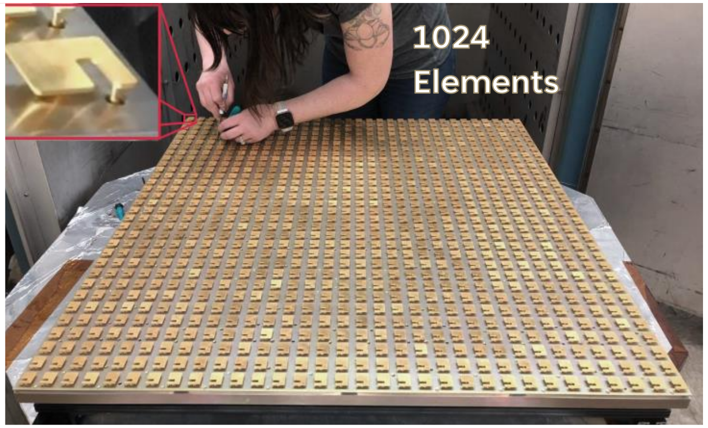
No Accumulated Charges

Mechanically Robust

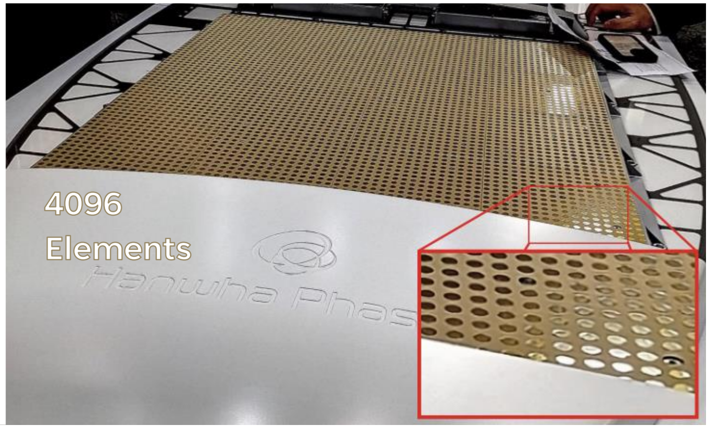
ANTENNA ARRAYS FOR SPACE APPLICATIONS (2/2)

Today

NASA JPL EUROPA LANDER (2018)



VIASAT/ESA (2021)



HANWHA PHASOR (2023)

Electrically Large Structures

Excessive number of unknowns for traditional methods → *Fast*

Densely Packed Elements

High-mutual coupling → *Full-wave*

Stringent Performance Req.

Strict error-controllability → *Accurate*

RESEARCH OBJECTIVES

Develop a **fast**, **accurate** yet **versatile** analysis method for aperiodic antenna arrays for space applications.

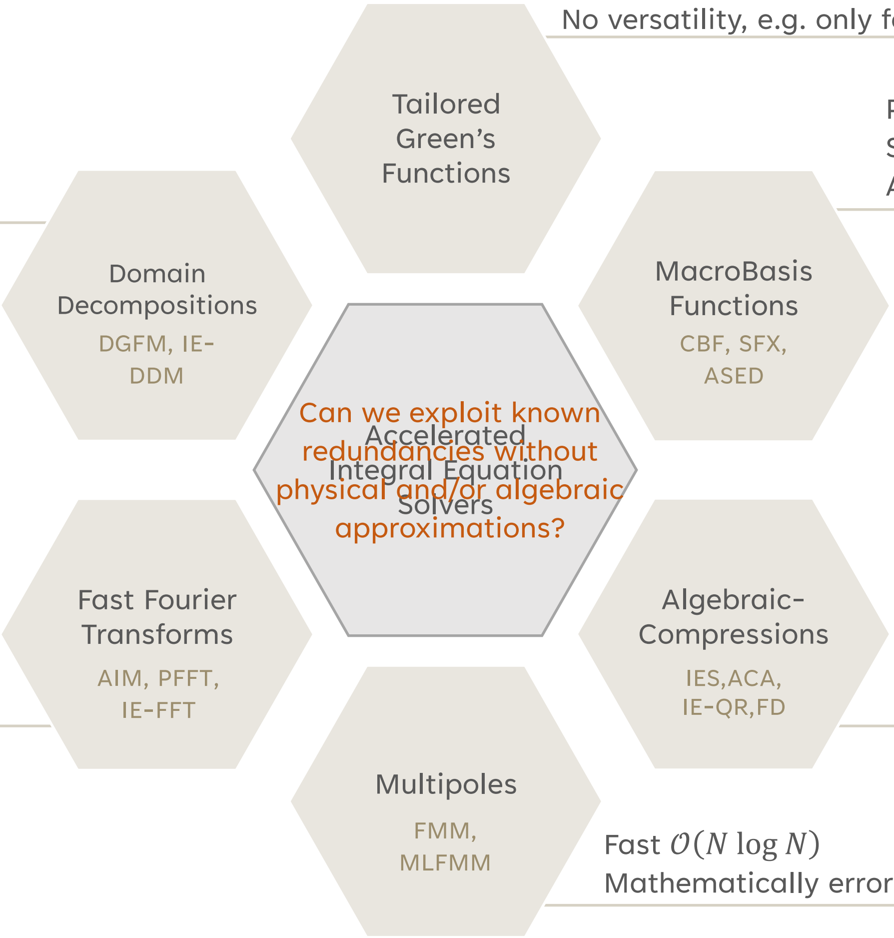
FAST	ACCURATE	VERSATILE
Asymptotic Complexity $\mathcal{O}(N \log N)^{\text{Fast}}$ Wall-clock time Less than existing method(s)	No method approximations whilst obtaining a faster method Error-controllability	Aperiodic Arrays Interconnecting Geometry Non-identical Elements Dielectric Substrates

TOWARDS NEW STATE-OF-THE-ART

Closed-form Green's Functions
No versatility, e.g. only for microstrip arrays

Division into and solution of local subproblems
Fast for well-separated array elements
Approximative element coupling

Problem specific aggregation of low-level basis functions
Smaller matrix; unknown computational complexity
Accuracy is dependent on validity of MBF's



Fast $\mathcal{O}(N \log N)$
Approximate: projection of BFs onto a regular grid

Clever grouping of BFs to achieve compression
Smaller matrix; $\mathcal{O}\left(N^{3/2}\right)$
Tradeoff: compression (speed & memory) vs. accuracy

Fast $\mathcal{O}(N \log N)$
Mathematically error-controllable approximations

Part I Introduction

Part II **Array Decomposition Method with Higher-Order Basis Functions**

- The Benefits of Higher-order Basis Functions
- Computational & Memory Complexity in Higher-order Schemes
- Numerical Validation Examples

Part III Connected & Aperiodic Array Antennas

Part IV Non-Identical & Non-All-Metal Arrays

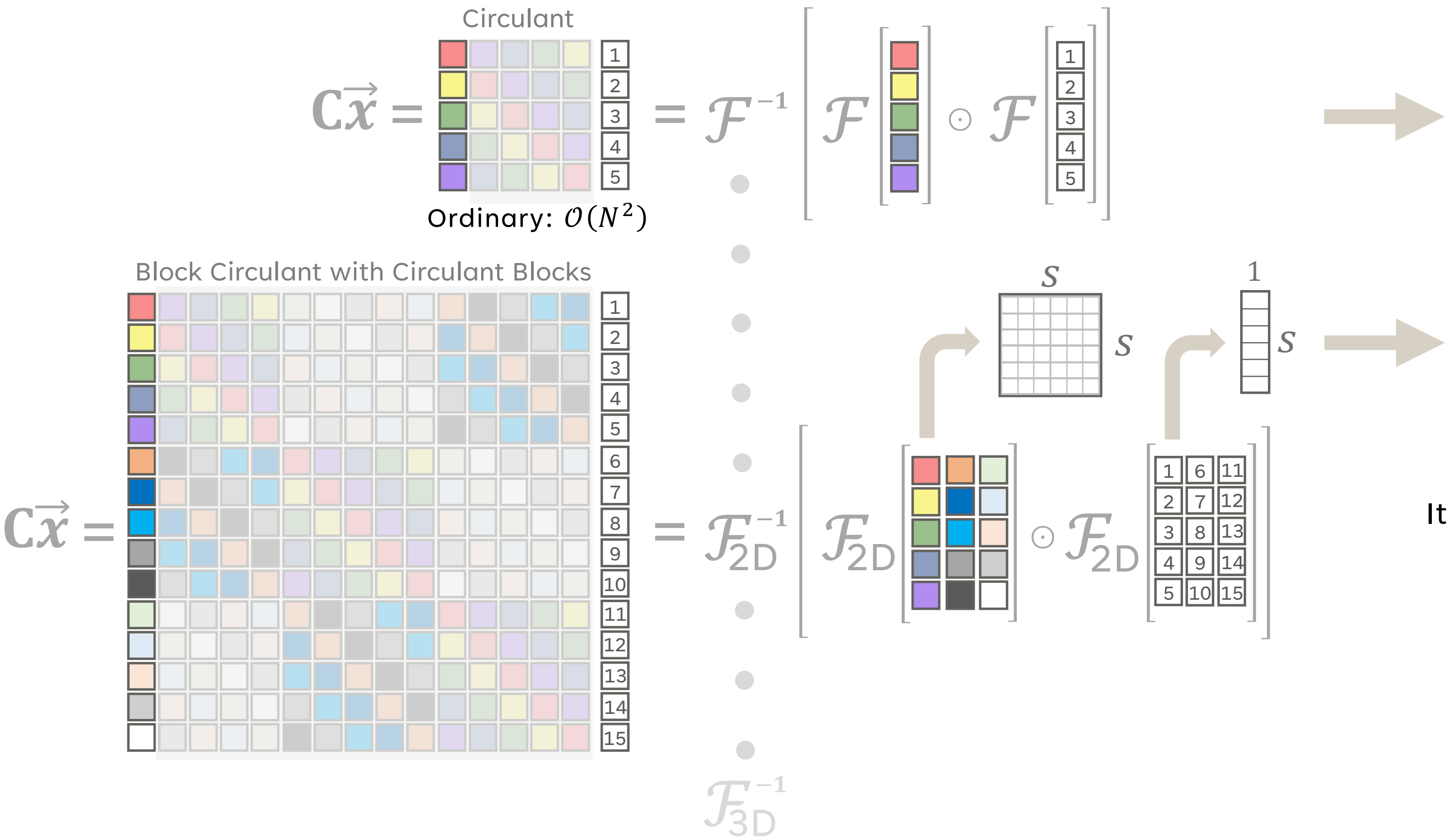
DISPOSITION

THE ARRAY DECOMPOSITION METHOD

KEY INGREDIENT 1

Ordinary Matrix-vector Product

Accelerated Matrix-vector product



Computational complexity

$\mathcal{O}(N \log N)$

Memory complexity

$\mathcal{O}(N)$

Memory & computational complexity becomes proportional to s^2

It is of interest to keep s as low as possible without loosing accuracy

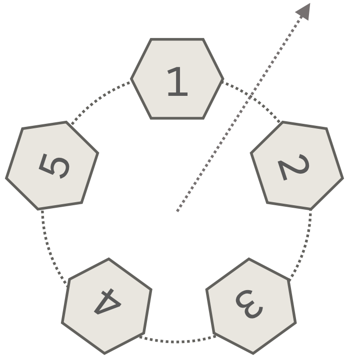
Higher-order basis functions

THE ARRAY DECOMPOSITION METHOD

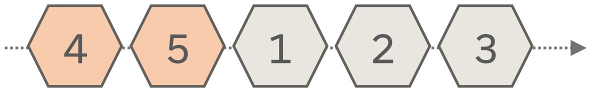
KEY INGREDIENT 2

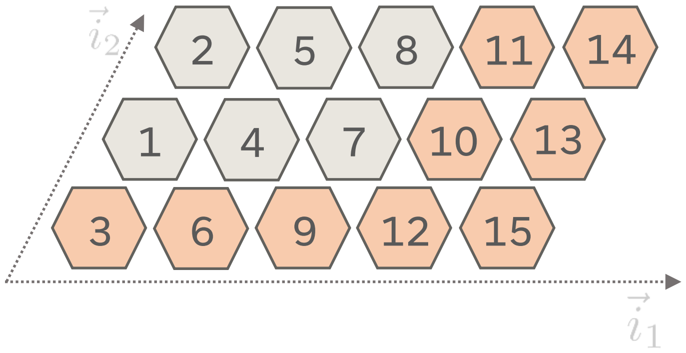
The Regular Lattice in MoM

Array Type



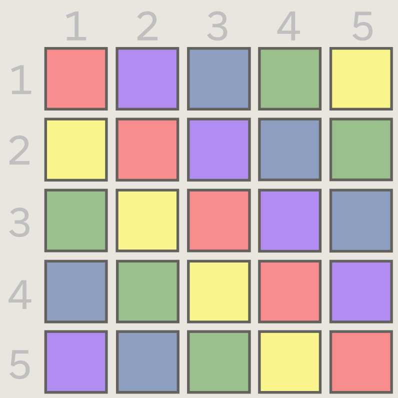
Virtual Extension



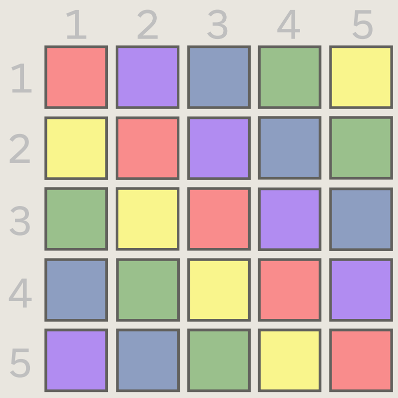


MoM Matrices

Block Circulant



Block Toeplitz → Block Circulant



Multi-Level Block Circulant

THE ARRAY DECOMPOSITION METHOD

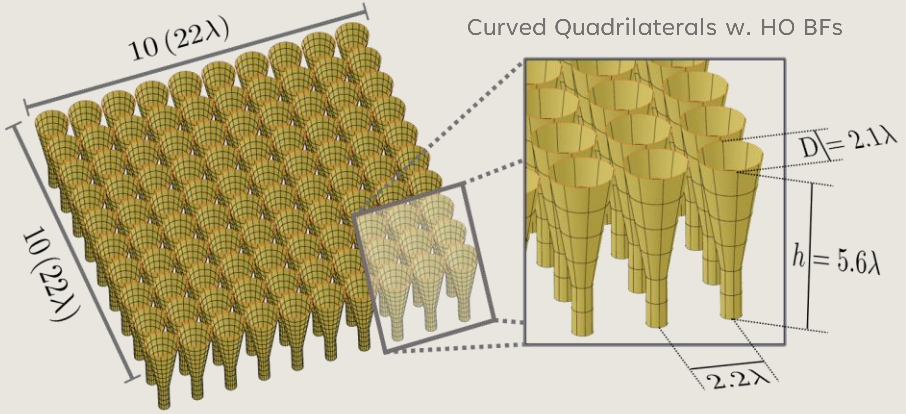
SUMMARY

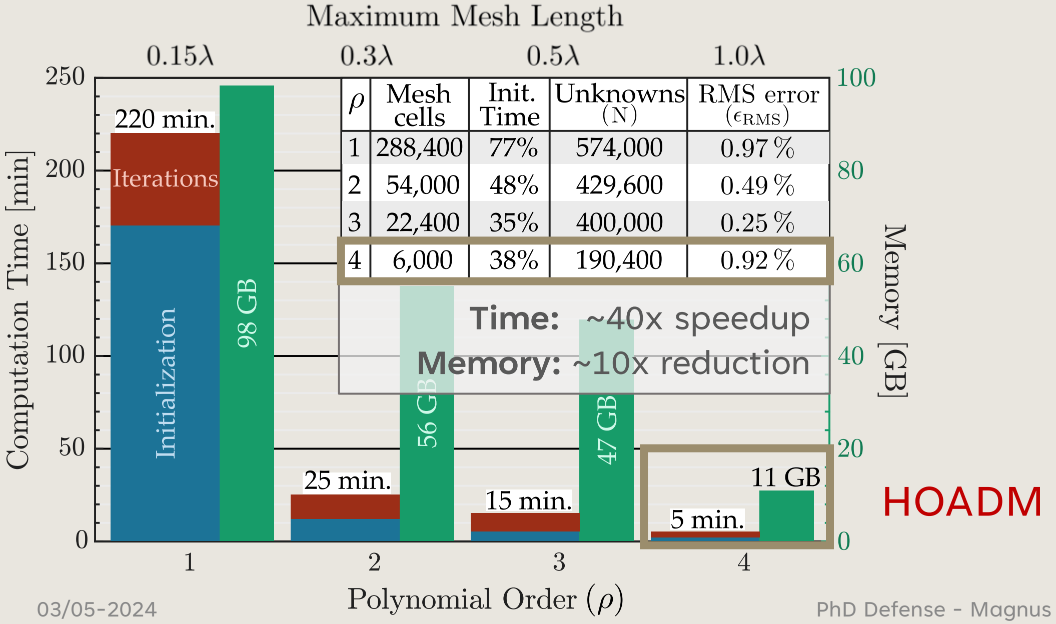
FAST ?	ACCURATE ✓	VERSATILE ?
Computational: $\mathcal{O}(s^2 T \log T)$ or $\mathcal{O}\left(sN \log \frac{N}{s}\right)$	Matrix-vector product is as exact as using full MoM Matrix	Aperiodic Arrays ✗
Memory: $\mathcal{O}(s^2 T)$ or $\mathcal{O}(sN)$		Interconnecting Geometry ✗
$N = s\prod_{i=1}^{d} n_i = sT$		Non-identical Elements ✗
Quadratic scaling		Dielectric Substrates ✗

How much can higher-order basis functions (BFs) help?

Hierarchical Modified Legendre BFs

THE BENEFITS OF HIGHER-ORDER BASIS FUNCTIONS





ρ	Mesh cells	Init. Time	Unknowns (N)	RMS error (ϵ_{RMS})
1	288,400	77%	574,000	0.97 %
2	54,000	48%	429,600	0.49 %
3	22,400	35%	400,000	0.25 %
4	6,000	38%	190,400	0.92 %

Horn array excited uniformly from circular waveguides with the TE_{11} fundamental mode at each port

We fix BF polynomial order ρ
Decrease mesh length until reaching < 1 % far-field error.

2π far-field hemi-sphere

$$\epsilon_{\mathrm{ERE}} = \sqrt{\frac{\sum_{i=1}^{N_s} \left|\boldsymbol{E}_{i,\mathrm{ref}} - \boldsymbol{E}_i\right|^2}{\sum_{i=1}^{N_s} \left|\boldsymbol{E}_{i,\mathrm{ref}}\right|^2}}$$

Full MoM Direct Solution

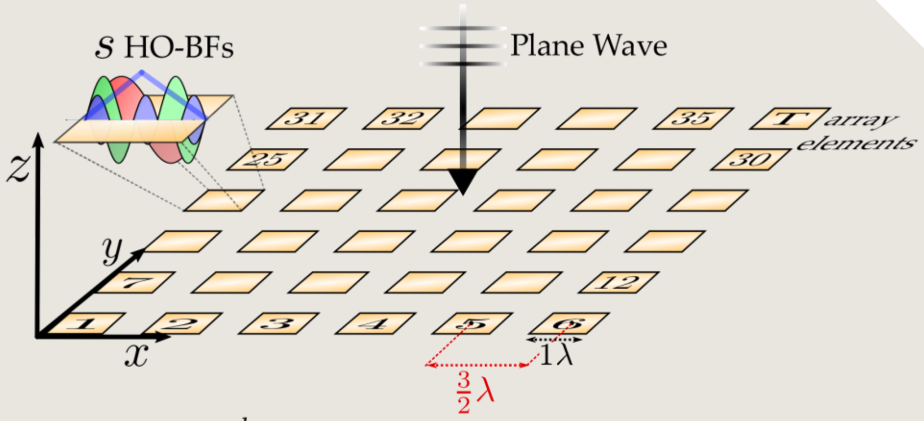
Initialization-time dominating for $\rho = 1$, due to the higher number of integrals.

The reduced number of mesh cells is the primary reason for the significant decrease in computation time for $\rho = 1 \rightarrow 2$

Small decrease in total number of unknowns from $\rho = 2 \rightarrow 3$ is due to the increase in accuracy caused by meshing constraints

COMPUTATIONAL & MEMORY COMPLEXITY IN HIGHER-ORDER SCHEMES

HOADM vs. HOMLFMM



$$N = s\prod_{i=1}^{d} n_i = sT$$

HOADM: $\mathcal{O}(s^2 T \log T)$

MLFMM: $\mathcal{O}(sT \log sT)$

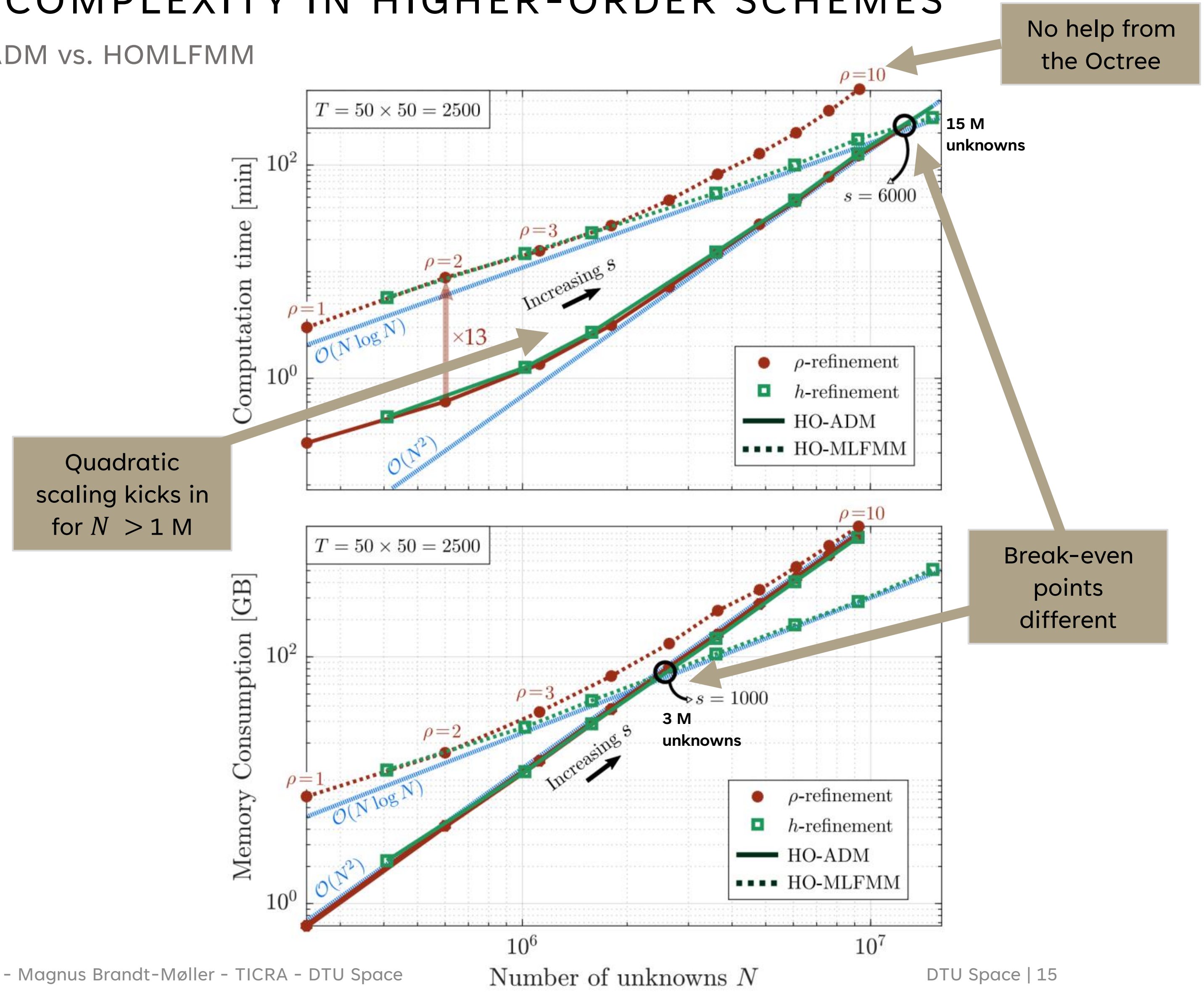
1. Varying the mesh length (h-refinement)
2. Varying the polynomial order (ρ-refinement)

Increasing s

3. Varying the total number of array elements T

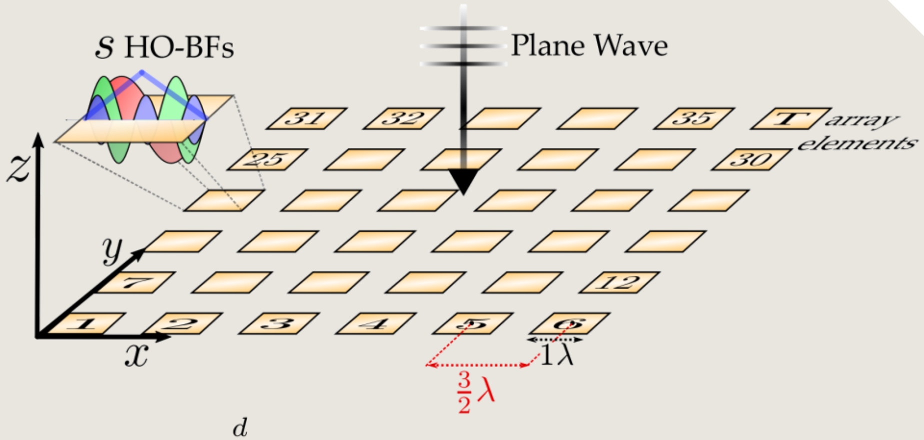
Computation time & Memory Consumption

- Basis Functions
- Unique matrix blocks
- Preconditioner
- Iterative Solution



COMPUTATIONAL & MEMORY COMPLEXITY IN HIGHER-ORDER SCHEMES

HOADM vs. HOMLFMM



$$N = s\prod_{i=1}^{d} n_i = sT$$

HOADM: $\mathcal{O}(s^2 T \log T)$
MLFMM: $\mathcal{O}(sT \log sT)$

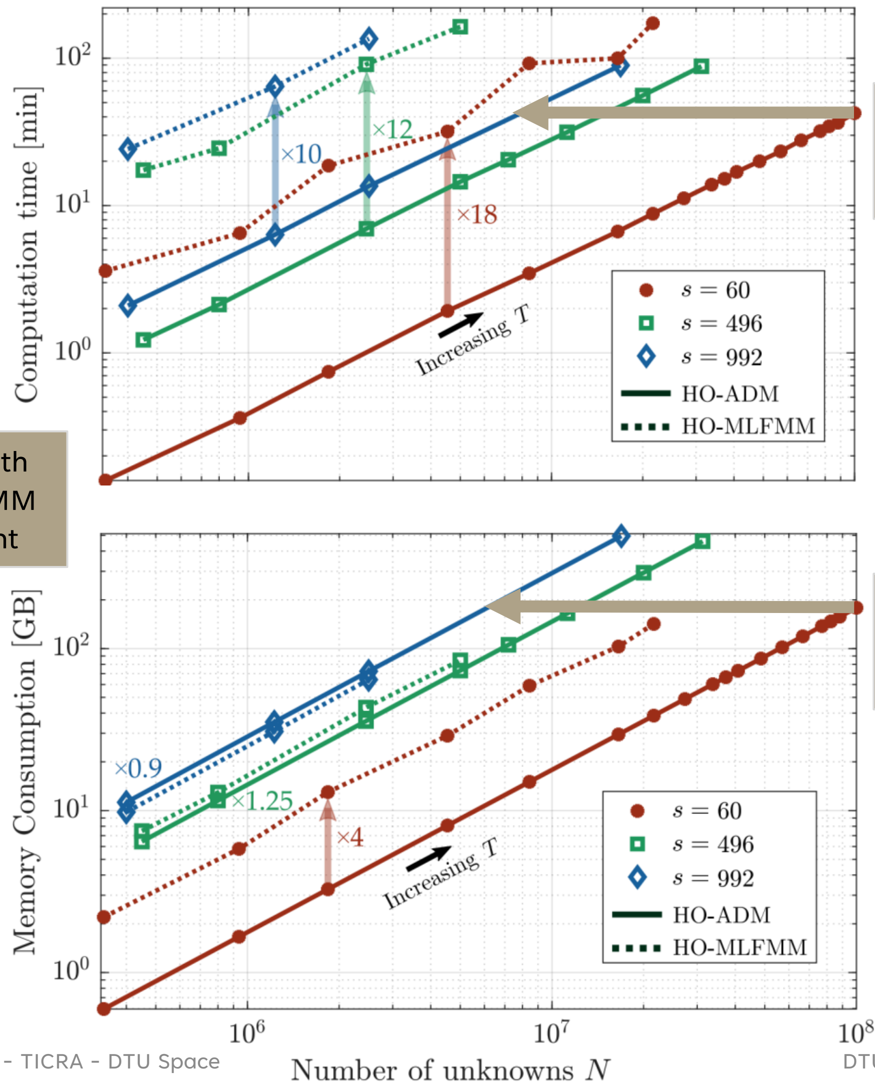
1. Varying the total number of array elements T } **Increasing T**

Main offering of HOADM
A lower complexity constant, enhancing time efficiency while maintaining the same accuracy as full MoM

Computation time & Memory Consumption

- Basis Functions
- Unique matrix blocks
- Preconditioner
- Iterative Solution

$\mathcal{O}(N \log N)$ for both HOADM and MLFMM when s is constant



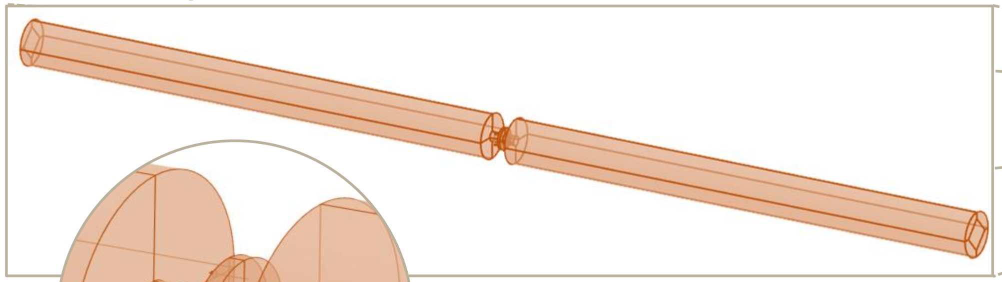
42 min
s = 60 → 992
N = 100 M → 8 M

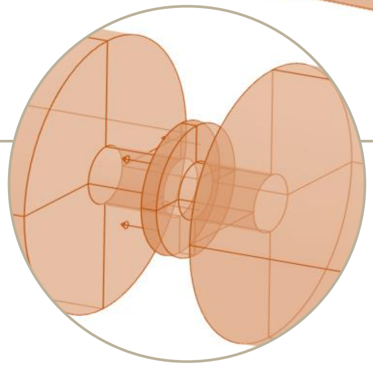
180 GB
s = 60 → 992
N = 100 M → 6 M

NUMERICAL VALIDATION EXAMPLE

BROADBAND SCATTERING PARAMETERS FOR 8X8 DIPOLE ARRAY

Single Dipole



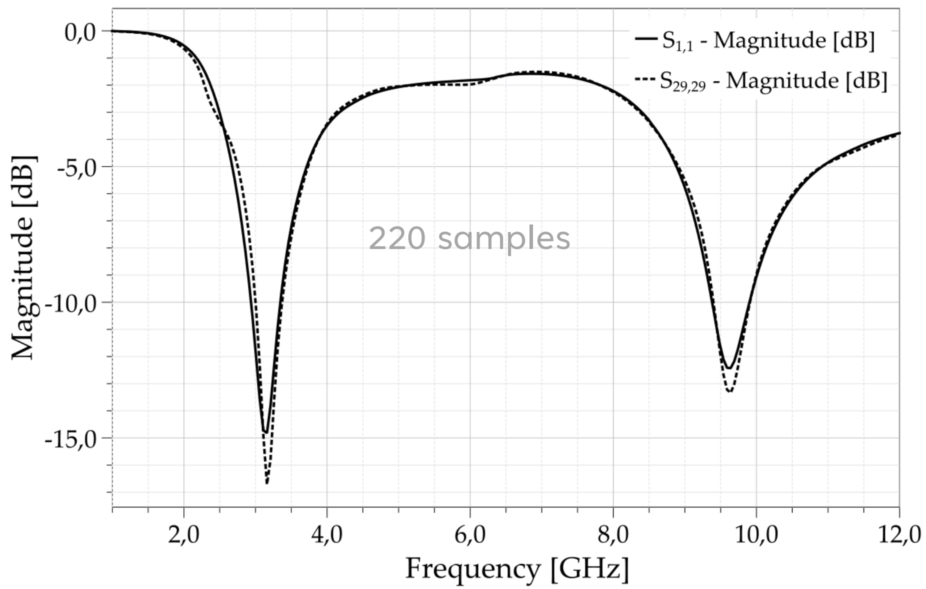


Waveguide Excitation

Fundamental TEM coaxial mode

$N = 27{,}456$ $s = 429$ $N_{\mathrm{RHS}} = 64$ $N_{\mathrm{freq}} = 220$	Computation Time	Peak Memory	Time Per Frequency
HO-ADM	59 min.	333 MB	16 s
MLFMM	12 hours	1.2 GB	3.3 min

Input Reflection Coefficients



Part I — Introduction

Part II — Array Decomposition Method with Higher-Order Basis Functions

Part III — **Connected & Aperiodic Array Antennas**

- Thinned & Sparse Arrays via Hidden Unknowns
- Connected Arrays via Auxiliary Unknowns & Discontinuous Galerkin
- Numerical Validation Examples

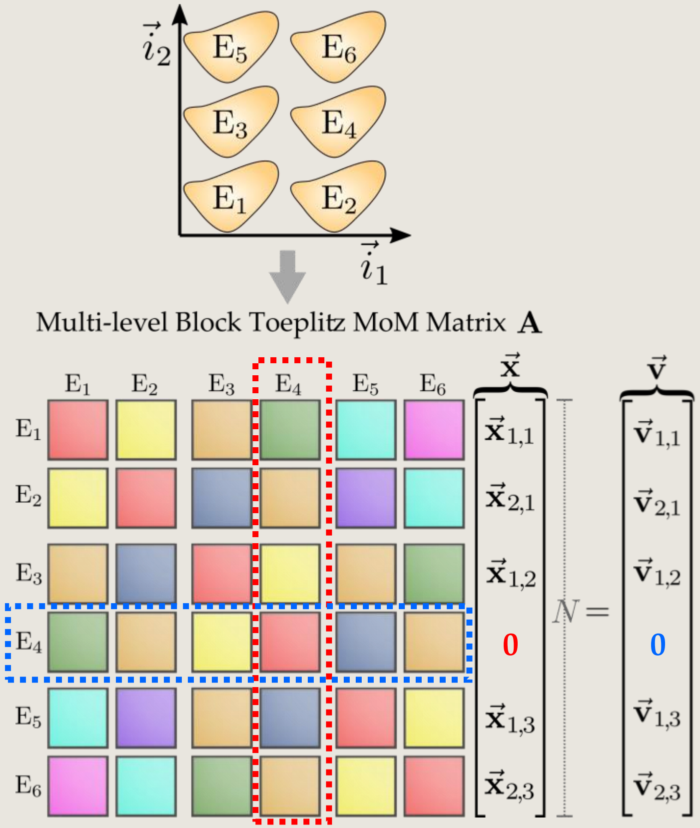
Part IV — Non-Identical & Non-All-Metal Arrays

DISPOSITION

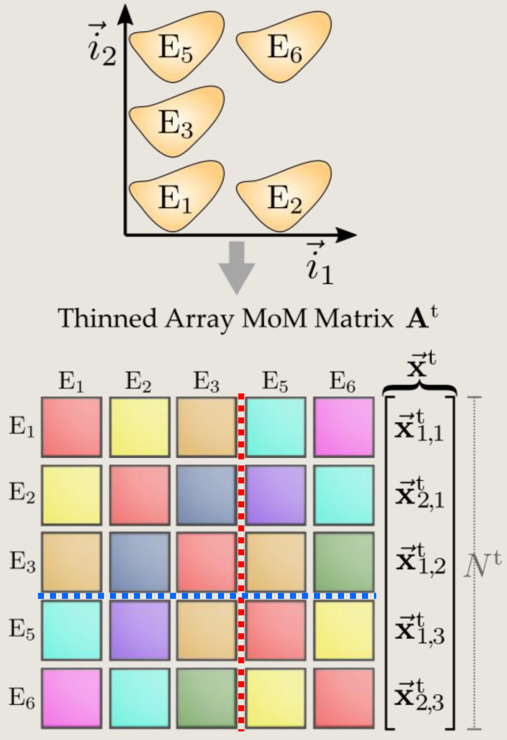
THINNED AND SPARSE ARRAYS

THE INTRODUCTION OF HIDDEN UNKNOWNS

Fully Populated Array

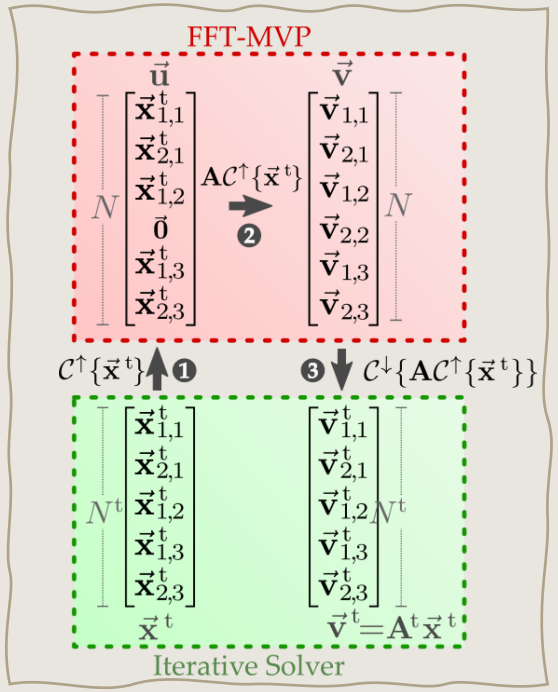


Thinned Array



Toeplitz Property Lost

Procedure of Hidden Unknowns

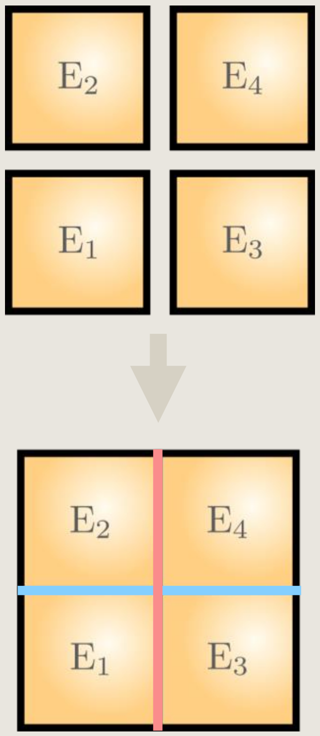


$$\vec{r}^{\,t} = \underbrace{\mathcal{C}^{\downarrow}\{\vec{b}\}}_{\vec{b}^{\,t}} - \underbrace{\mathcal{C}^{\downarrow}\{\mathbf{A}\mathcal{C}^{\uparrow}\{\vec{x}^{\,t}\}\}}_{\mathbf{A}^{t}\vec{x}^{\,t}}$$

CONNECTED ARRAYS

DISCONTINUOUS GALERKIN AND AUXILIARY UNKNOWNS

Connecting Array Elements



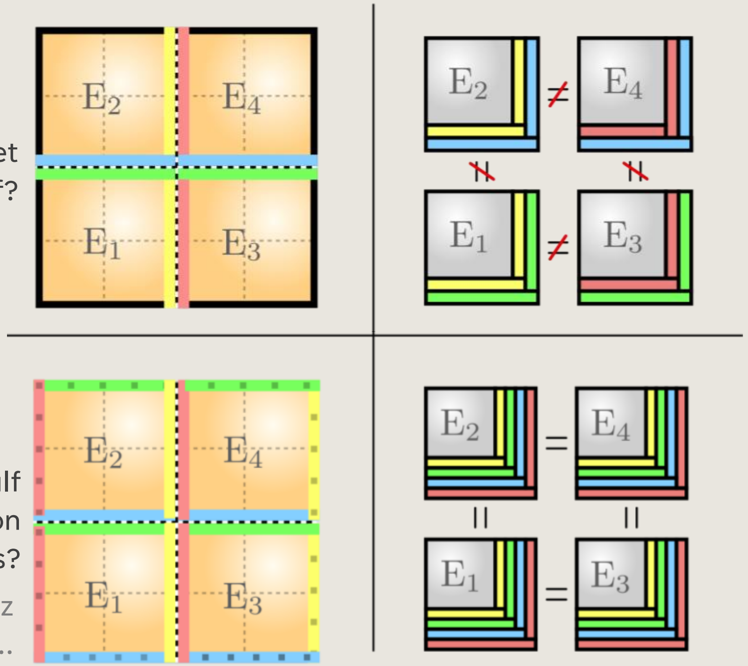
Shared Doublet BFs
||
Toeplitz Property Lost

Regaining the Toeplitz Property

Split doublet BFs in half?

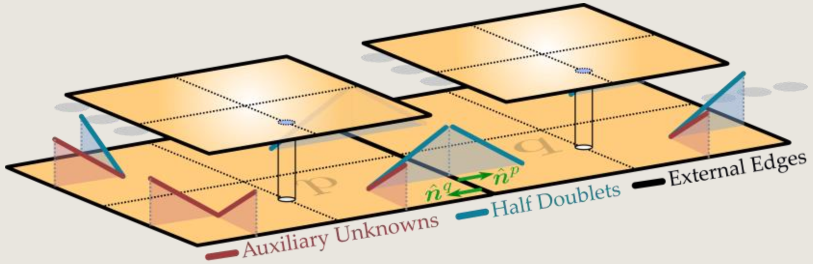
Place half doublet BFs on all edges?

Yes, Toeplitz but…



1. No longer same electromagnetic configuration
2. What about current continuity?

Enforcing Current Continuity



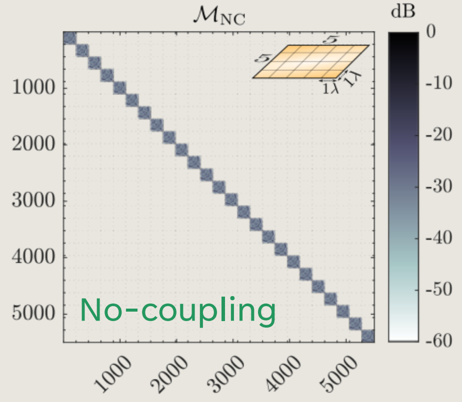
1. Hide Auxiliary Unknowns from iterative solver
2. Use Discontinuous Galerkin to allow for conduction currents between elements
3. Omit interior stabilization function:
$$\mathcal{I}_{\mathrm{IP}}(\beta) = \frac{\beta}{k^2}\int_{\mathcal{C}_{pq}} [\hat{\boldsymbol{n}}^p\cdot \vec{f}_t^m(\vec{r})][\hat{\boldsymbol{n}}^q\cdot \vec{f}_b^n(\vec{r})]d\vec{r}$$
4. MoM Matrix condition number is now high…

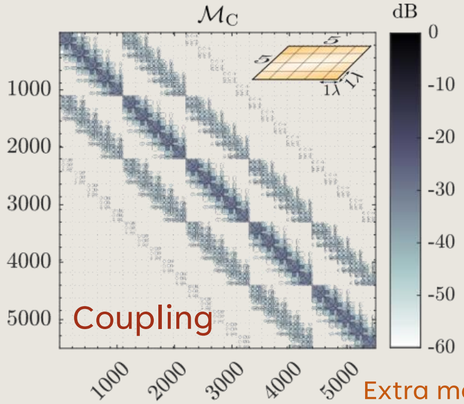
CONNECTED ARRAYS

REQUIRED PRECONDITIONING

Block-diagonal Preconditioning

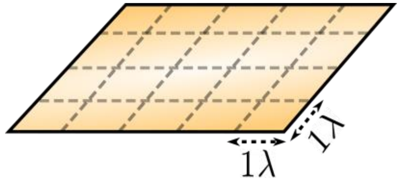
$$\mathcal{M}^{-1}\mathbf{A}\vec{x} = \mathcal{M}^{-1}\vec{b}$$



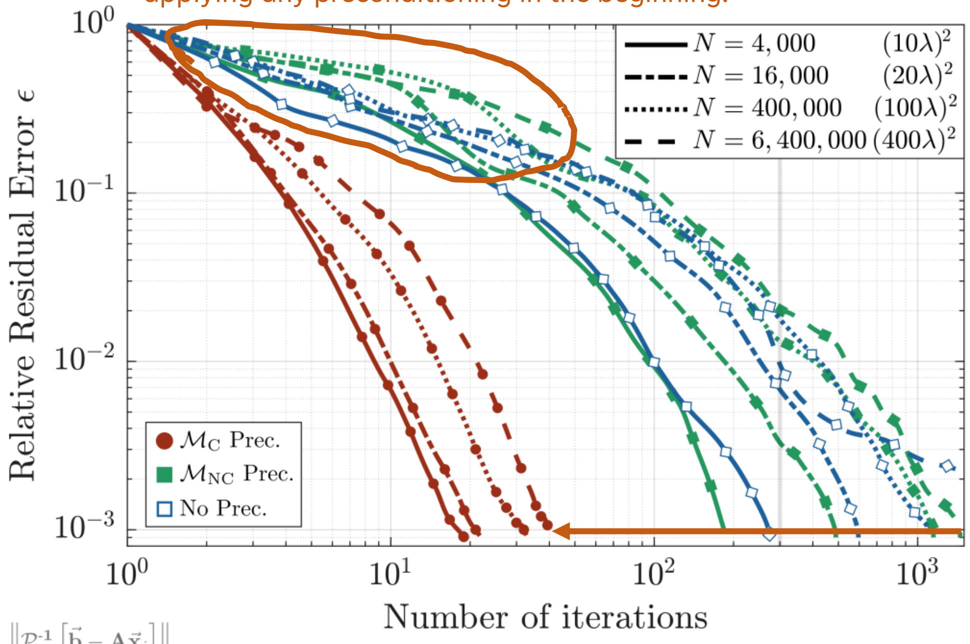


Extra memory $\mathcal{O}(N)$

Simple PEC Plate Modelled as an Array



No-coupling preconditioner worse than not applying any preconditioning in the beginning.



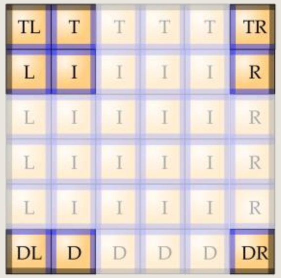
1448 iterations
↓
41 iterations

$$\epsilon = \frac{\left\|\mathcal{P}^{-1}\left[\vec{\mathbf{b}} - \mathbf{A}\vec{\mathbf{x}}_i\right]\right\|_2}{\left\|\mathcal{P}^{-1}\vec{\mathbf{b}}\right\|_2}$$

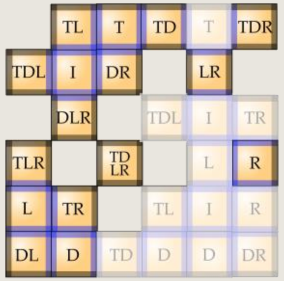
Constant Memory Preconditioner

1 block
Unconnected

9 blocks
Connected



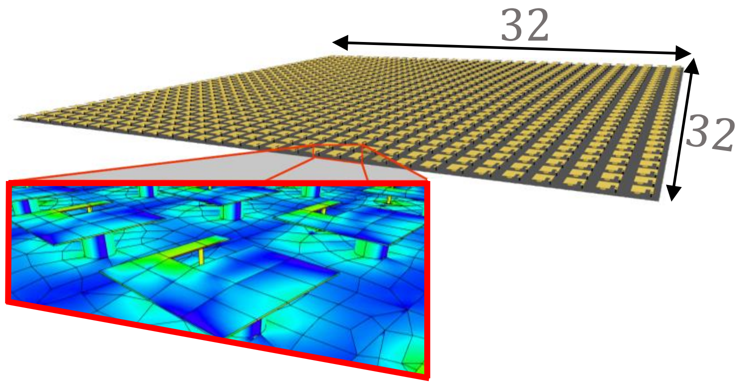
16 blocks
Connected & Thinned



Intel® i7-13850HX 20 Core CPU
@ 2.60GHz 128 GB RAM

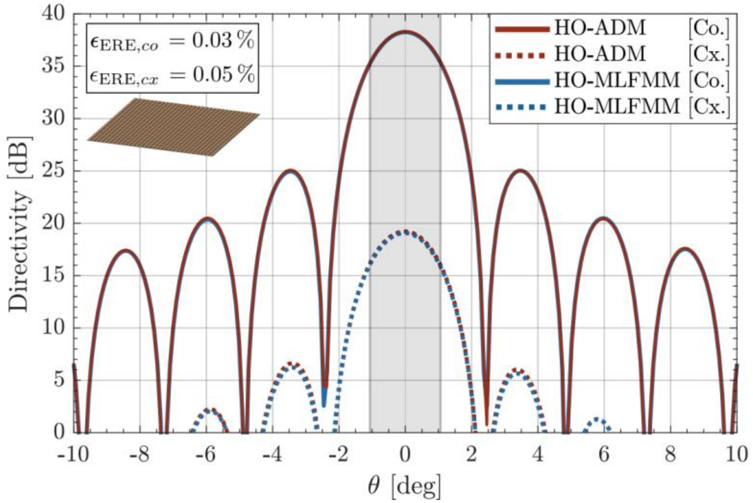
NUMERICAL VALIDATION EXAMPLE

ALL-METAL 32X32 DUAL BAND RHCP ARRAY FOR THE EUROPA LANDER MISSION

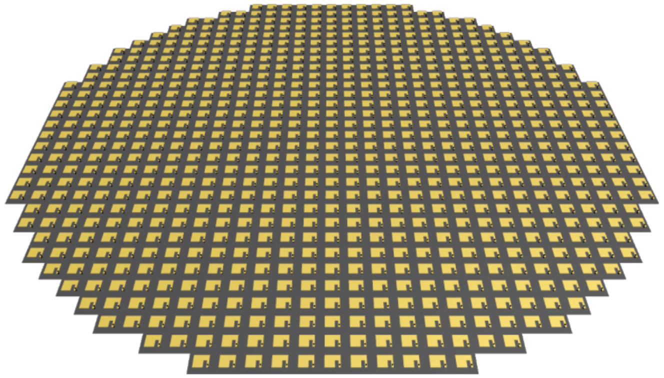


Recent optimizations of my code
2 min 57 s

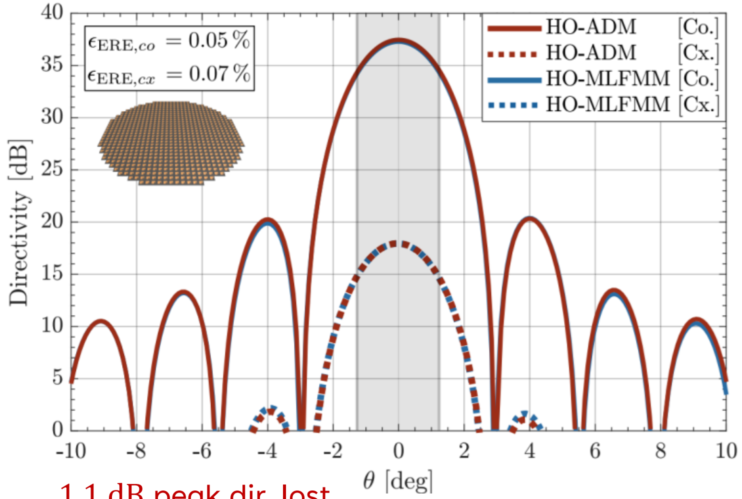
$T = 1024$ $s \approx 1000$	Computation Time	Peak Memory	Number of iterations	Time per Iteration
HOADM $N \approx 1$ M	6 min 18 s	28.2 GB	481	0.40 s
MLFMM $N \approx 975{,}000$	1 hour 2 min	25.1 GB	540	3.94 s



793-elements (23 % thinned)



$T = 1024$ $s \approx 1000$	Computation Time	Peak Memory	Number of iterations	Time per Iteration
HOADM $N \approx 1$ M	6 min 18 s	28.2 GB	506	0.38 s
MLFMM $N \approx 755{,}000$	**57 min**	**22.5 GB**	567	3.81 s



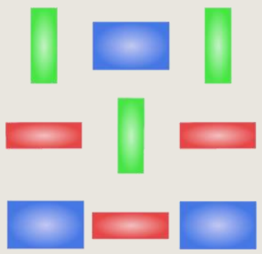
1.1 dB peak dir. lost
0.2 dB beamwidth lost

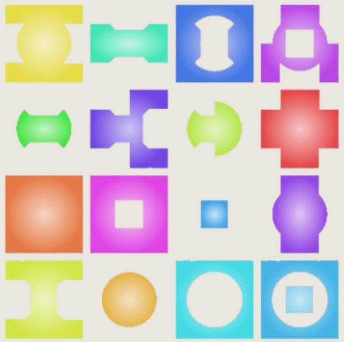
DISPOSITION

NON-IDENTICAL ELEMENTS

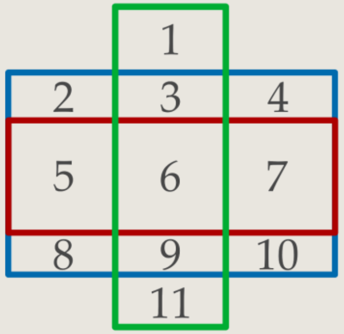
THE INTRODUCTION OF THE SUPER UNIT CELL

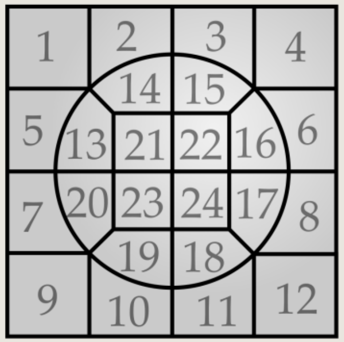
Non-Identical Element Array



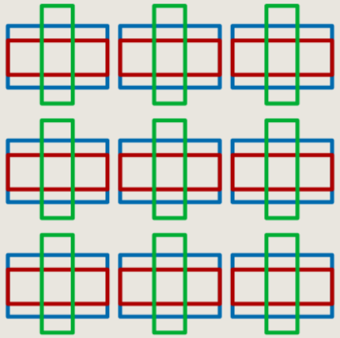


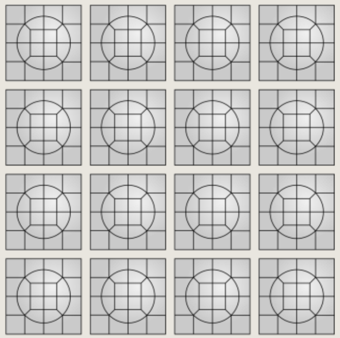
Required Super Unit Cell (SUC)





Identical Element Array



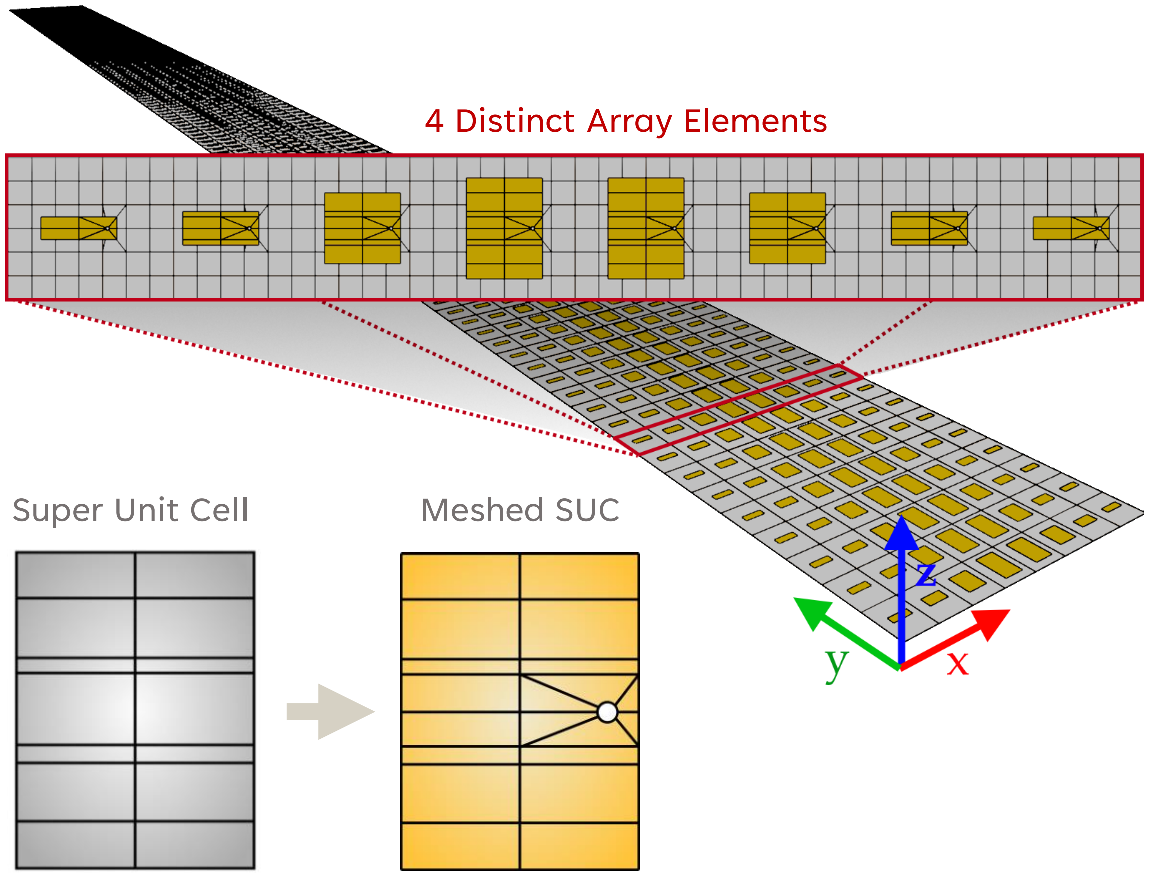


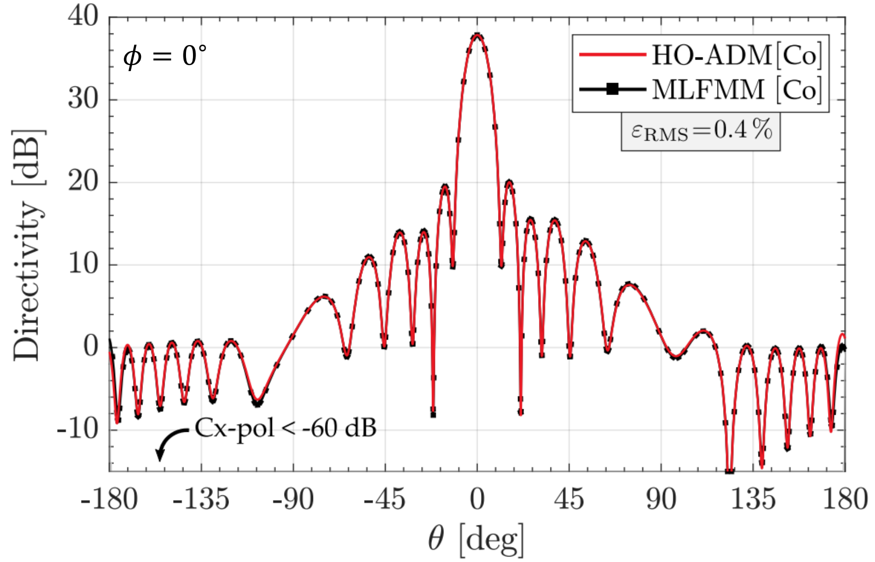
Procedure

1. Element Outlines are Superimposed (union)
2. Meshing of Super Unit Cell based on electrical size
3. Identification of Hidden Unknowns for each element
4. No need to recompute matrices for different configurations

NUMERICAL VALIDATION EXAMPLE

1024 ELEMENT PATCH ARRAY WITH NON-IDENTICAL ELEMENTS



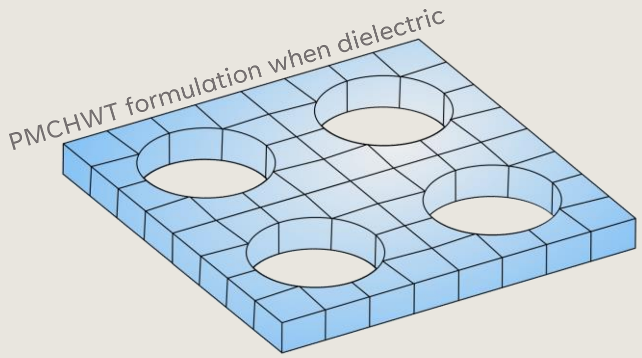


$T = 1024$ $s = 496$	Computation Time	Memory
HO-ADM $N = 507{,}904$	3 min.	7.9 GB
MLFMM $N = 440{,}736$	29 min.	7.2 GB
MoM	-	1443 GB

FINITE THICKNESS ARRAYS AND DIELECTRIC SUBSTRATES

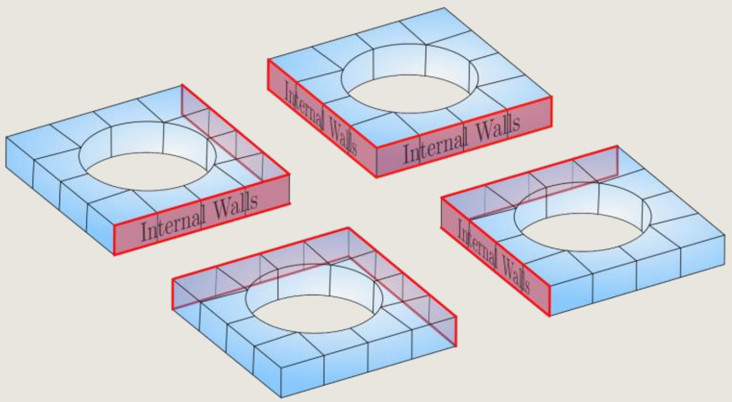
INTERNAL WALLS & EQUIVALENT CURRENTS

Closed Surface Array Structure



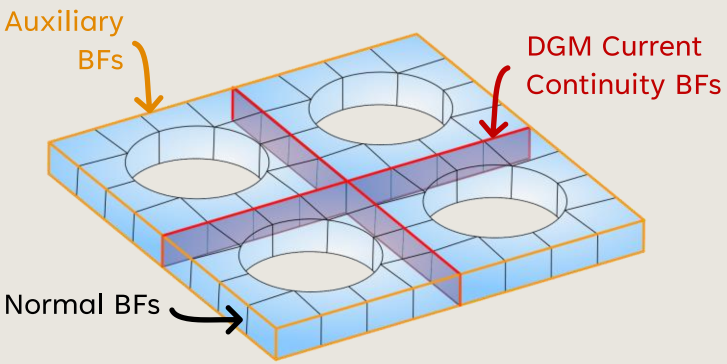
1. Topologically identical array elements
2. Electromagnetically different boundary conditions
3. Toeplitz property cannot be obtained

Placement of Internal Walls



1. Equivalent currents on internal walls radiate into the background medium
2. Now electromagnetically identical array elements
3. No longer the same physical structure

Identical-element Array Structure

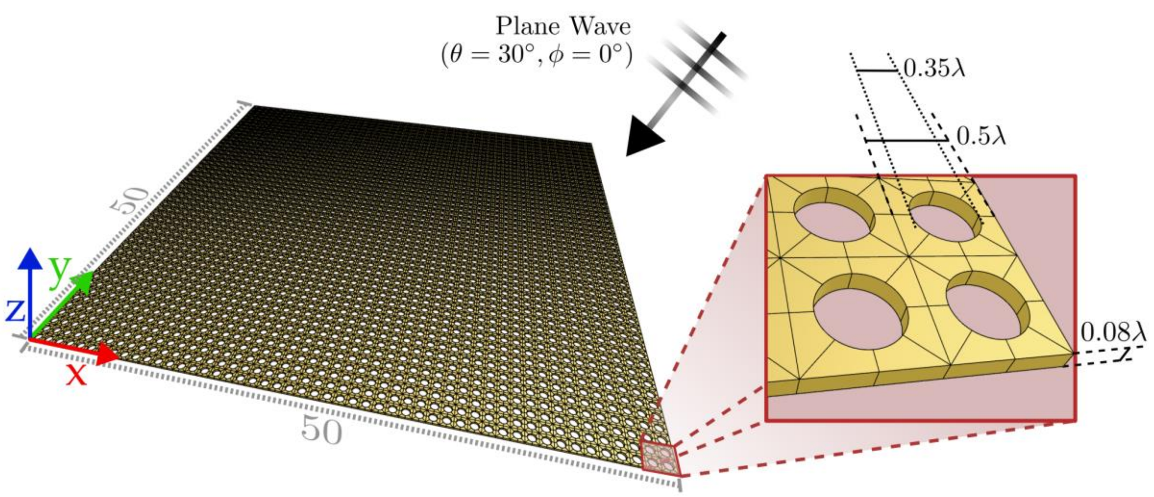


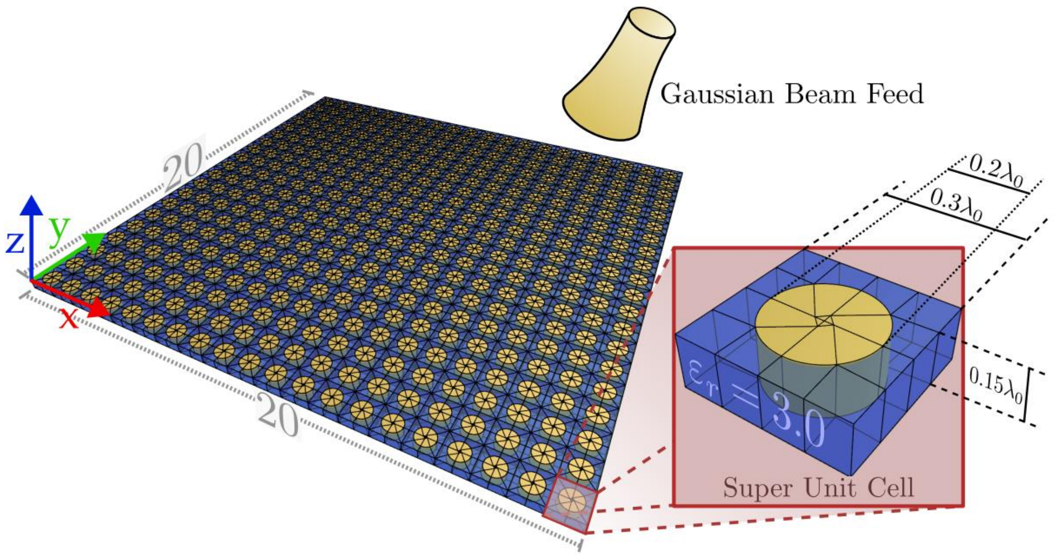
1. Half doublets are placed on those edges across which currents should flow
2. Auxiliary half doublets are placed on outer array elements
3. Leveraging the hidden unknowns technique

NUMERICAL VALIDATION EXAMPLES

ARRAYS WITH FINITE THICKNESS AND DIELECTRIC MATERIAL

Intel® i7-9850H 6 Core CPU @ 2.60GHz 32 GB RAM





	$T = 2500$ $s = 456$	Computation Time	Memory	Iterations	0.3 % far-field error
77% more unknowns	HO-ADM $N = 1{,}140{,}000$	4 min.	15.2 GB	$\boxed{165}$	Preconditioner Struggles
	MLFMM $N = 646{,}400$	68 min.	18.1 GB	58	

	$T = 400$ $s = 1056$	Computation Time	Memory	Iterations	0.7 % far-field error
58% more unknowns	HO-ADM $N = 422{,}400$	11 min.	12.6 GB	$\boxed{758}$	Preconditioner Not Working Properly
	MLFMM $N = 267{,}520$	1 hours 39 min	13.7 GB	249	

CONCLUSIONS & OUTLOOK

A fast, accurate yet versatile computational analysis technique has been developed for electrically large arrays for space applications

Fast
In many practical cases more than an order of magnitude faster than MLFMM.
Higher-order basis functions made ADM applicable for antenna arrays in practice.

Accurate
Full-wave and as rigorous as MoM. No method parameters to tune, hence no “method convergence”

Versatile
Aperiodic Arrays, Connected Arrays, Non-identical Arrays and Dielectric Substrates.

While generalized fast full-wave methods can solve any array-type, it can be worth it to exploit known redundancies.

Hybridization
Investigate the possibility of incorporating ADM in a DDM technique together with e.g. MLFMM or Fast Direct Solvers

Compression
Error-controllable compression of Fourier coefficients or incorporation of well-known ACA or rSVD techniques.

Preconditioning for Dielectrics
- Devise a better preconditioning strategy in case of dielectric substrates
- Dielectrics with non-identical elements.

and more...

THANK YOU FOR YOUR ATTENTION

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Dissertation PDF

ORIGINAL CONTRIBUTIONS

- **Development of HO-ADM for Regular Arrays**
  The existing boundary integral part of the Array Decomposition Method (ADM) has, for the first time, been combined with higher-order basis functions. Numerical tests have verified that this combination, denoted HO-ADM, leads to more than an order of magnitude lower memory consumption and faster computation times compared to ADM employing traditional first-order basis functions. Furthermore, for the first time, higher-order convergence has been demonstrated for ADM in combination with hierarchical higher-order basis functions.
- **Arrays with Interconnecting Geometry**
  The HO-ADM has been extended to allow for the flow of electric conduction currents between array elements. This advancement was realized using a unique application of the established Discontinuous Galerkin Method. As a result, the corresponding Method of Moments (MoM) matrix maintains its block-Toeplitz property and more importantly ensures that the HO-ADM continues to benefit from its FFT-accelerated matrix-vector product. The effective solution is made possible by realizing an asymptotically constant-memory preconditioner applicable for HO-ADM.
- **Aperiodic Arrays with Non-identical Elements**
  The functionality of HOADM has been expanded to accommodate non-uniform arrays, such as sparse or thinned arrays, through selective truncation of the Krylov subspace in the iterative solution process. Furthermore, with the introduction of a super unit-cell (SUC) strategy, the method can handle non-identical elements while maintaining computational effciency across various element configurations.
- **Integration with Dielectric Substrates**
  The constrained Krylov subspace idea is used together with special geometrical meshing to allow the HO-ADM to handle arrays with a dielectric substrate, broadening the solver’s applicability across a myriad of array designs.